# Diffusion of optical pulses in dispersion-shifted randomly birefringent optical fibers

Pavel M. Lushnikov[1,2]

[1] *Theoretical Division, Los Alamos National Laboratory,*

*MS-B213, Los Alamos, New Mexico, 87545*

[2] *Landau Institute for Theoretical Physics,*

*Kosygin St., 2, Moscow, 119334, Russia* *

## Abstract

An effect of polarization-mode dispersion, nonlinearity and random variation of dispersion along an optical fiber on a pulse propagation in a randomly birefringent dispersion-shifted optical fiber with zero average dispersion is studied. An averaged pulse width is shown analytically to diffuse with propagation distance for arbitrary strong pulse amplitude. It is found that optical fiber nonlinearity can not change qualitatively a diffusion of pulse width but can only modify a diffusion law which means that a root mean square pulse width grows at least as a linear function of the propagation distance.

Polarization-mode dispersion (PMD), which is a pulse broadening caused by random variation of optical fiber birefringence, has recently become a major drawback in the development of new high-bit-rate optical communication systems [1, 2, 3, 4, 5, 6, 7]. Another effect, which limits bit-rate capacity, is pulse broadening caused by group-velocity dispersion (GVD). Use of a dispersion-shifted fiber with zero average GVD can reduce this effect, however, in such fibers GVD inevitably fluctuates around zero along the propagation direction [8, 9] and hence pulse broadening still occurs [10, 11]. Nonlinearity in optical fibers results in the coupling of both PMD and GVD effects, so in general they can not be studied separately in contrast to linear case. Linear PMD was first studied in Refs. [1, 4, 5] while nonlinear PMD was addressed in numerical experiments [2] and analytical studies based on a perturbation expansions around soliton solutions of deterministic equations [3, 12, 13, 14, 15]. An effect of random variation of GVD was studied in Refs. [10, 11, 16, 17, 18]. Here an exact analytical (nonperturbartive) theory is developed for the case of fiber with random birefringence and random GVD with zero mean and arbitrarily strong nonlinearity (arbitrary pulse amplitude). No assumption like closeness to any type of soliton solution is necessary for the results of this Article to be valid. The main result is that a statistical average (over random variation of fiber parameters) of root mean square pulse width $T_{RMS}$ grows with distance at least as a linear function of propagation distance. This means that random diffusion of optical pulse width can not be prevented by an arbitrarily strong nonlinearity. It is shown that random diffusion fundamentally limits the bit-rate capacity of an optical fiber.

Neglecting second-order GVD (dispersion slope) effects, stimulated Raman scattering and Brillouin scattering, the propagation of optical pulses in birefringent optical fibers is described by the two-component vector nonlinear Schrödinger equation (VNLS)[3, 14, 19, 20]

$$i\partial_z\eta_\alpha + \sum_{\beta=1}^{2}\triangle_{\alpha\beta}(z)\eta_\beta + i\sum_{\beta=1}^{2}\tilde{m}_{\alpha\beta}(z)\partial_t\eta_\beta + d(z)\partial_t^2\eta_\alpha$$
$$+\sigma(z)\tilde{N}_\alpha(\eta) = iG(z)\eta_\alpha, \tag{1}$$

where $z$ is the propagation distance along an optical fiber, $\eta_1$ and $\eta_2$ correspond to the complex amplitudes of two orthogonal linear polarizations, $t \equiv \tau - z/c_l$ is the retarded time and $\tau$ is the physical time, $c_l$ is the speed of light, and $d(z)$ is the dispersion, which is related to first-order GVD $\beta_2$ as $d(z) = -\frac{1}{2}\beta_2(z)$. The right hand side (rhs) of Eq. (1) describes linear losses and amplifiers, $G(z) \equiv \left(-\gamma + [\exp{(z_a\gamma)} - 1]\Sigma_{k=1}^{N}\delta(z - z_k)\right)$, $\sigma = (2\pi n_2)/(\lambda_0 A_{eff})$

is the nonlinear coefficient, $n_2$ is the nonlinear refractive index, $\lambda_0 = 1.55\mu m$ is the carrier wavelength, $A_{eff}$ is the effective fiber area, $z_k = kz_a$ ($k = 1, \ldots, N$) are the amplifier locations, $z_a$ is the amplifier spacing, and $\gamma$ is the loss coefficient. Distributed amplification can be also included by adding $z$-dependence into $\gamma$. Properties of fiber can be different along optical line, e.g. $A_{eff}$ could be different if line consists of several pieces of fiber with different cross section, and, respectively, coefficient $\sigma$ generally depends on $z$. In a similar way, all parameters of fiber, like $d(z)$ also depend on $z$.

The self-conjugated matrices $\hat{\triangle}(z)$ and $\hat{\tilde{m}}(z)$ describe, respectively, the differences in wave vectors and the anisotropy of the group velocities of the two modes corresponding to the two different polarizations. Both matrces $\hat{\triangle}$ and $\hat{\tilde{m}}$ are made traceless. The trace of the matrix $\hat{\triangle}$ is excluded by a phase transformation $\tilde{\eta} \to \eta \exp(i\phi_0 z)$. The trace of the matrix $\hat{\tilde{m}}$ is zero because Eq. (3) is written in a frame moving with average group velocity (note that group velocity is generally $z$-dependent). It is assumed in Eq. (3) that the dispersion $d(z)$ and nonlinearity are isotropic because their anisotropy is usually negligible in optical fibers. Vector $\tilde{\mathbf{N}} = (\tilde{N}_1, \tilde{N}_2)^T$, which represents the contribution of Kerr nonlinearity, is given by:

$$
\begin{aligned}
\tilde{N}_1(\Psi) &= \left[ (|\Psi_1|^2 + \frac{2}{3}|\Psi_2|^2)\Psi_1 + \frac{1}{3}\Psi_2^2\Psi_1^* \right], \\
\tilde{N}_2(\Psi) &= \left[ (\frac{2}{3}|\Psi_1|^2 + |\Psi_2|^2)\Psi_2 + \frac{1}{3}\Psi_1^2\Psi_2^* \right]
\end{aligned}
\tag{2}
$$

(see [3, 19]).

The change of variables $\xi = \eta e^{-\int_0^z G(z')dz'}$ (see e.g. Refs. [21, 22]) removes rhs of Eq. (1) and gives:

$$
\begin{aligned}
i\partial_z\xi_\alpha + \sum_{\beta=1}^{2} \triangle_{\alpha\beta}(z)\xi_\beta + i\sum_{\beta=1}^{2} \tilde{m}_{\alpha\beta}(z)\partial_t\xi_\beta \\
+ d(z)\partial_t^2\xi_\alpha + c(z)\tilde{N}_\alpha(\xi) = 0,
\end{aligned}
\tag{3}
$$

where $c(z) \equiv \sigma(z) \exp\left( 2\int_0^z G(z')dz' \right)$. Thus all linear fiber losses and amplifications are included into coefficient $c(z)$.

The isotropic case, which corresponds to zero matrices $\hat{\triangle} = \hat{\tilde{m}} = \hat{0}$, allows a solution of Eq. (3) with constant polarization, e.g. $\xi_1 \neq 0, \xi_2 = 0$. Components of matrices $\hat{\triangle}$ and $\hat{\tilde{m}}$ fluctuate strongly as functions of distance $z$. Fluctuations correspond to violation of circular symmetry of the fiber. The matrices $\hat{\triangle}$ and $\hat{\tilde{m}}$ change in optical fibers with time on a scale of few hours because of environmental fluctuations, however, for typical optical pulse duration

(10ps), one can consider $\hat{\triangle}$ and $\hat{\tilde{m}}$ as functions of $z$ only. It means that disorder is frozen in the fiber. The matrix $\hat{\triangle}$ gives the leading order contribution in Eq. (3) because a typical beat length $z_{beat}$ (typical length at which a relative phase shift between two polarizations caused by $\hat{\triangle}$ becomes $\sim \pi$) is $\sim 20m$ in optical fibers [6, 12]. Contribution of all other terms in Eq. (3) are essential on a scale of the order of $10km$ and larger. Thus, it is convenient to introduce a slow variable $\Psi$ as

$$\eta_\alpha = \sum_{\beta=1}^{2} T_{\alpha\beta}(z)\Psi_\beta, \tag{4}$$

where the unitary matrix $\hat{T}(z)$ is given by the solution of the matrix equation:

$$\frac{dT_{\alpha\beta}}{dz} = i \sum_{\delta=1}^{2} \triangle_{\alpha\delta}(z)T_{\delta\beta}, \quad T_{\alpha\beta}(0) = \hat{I}, \tag{5}$$

where $\hat{I}$ is the identity matrix.

The slow variable $\Psi$ changes on the scale $\sim 10km$ because all fast dependence on $\hat{\triangle}$ is included into $\hat{T}$. Thus Eq. (3) can be averaged over distance much larger than $z_{beat}$, but still much smaller than $10km$ [3, 6, 14, 15]. Here we use the simplest possible form of an averaged equation [3, 14, 15]:

$$i\partial_z\Psi_\alpha + i \sum_{\beta=1}^{2} m_{\alpha\beta}(z)\partial_t\Psi_\beta + d(z)\partial_t^2\Psi_\alpha + N_\alpha = 0, \tag{6}$$

where $\hat{m}(z) = \hat{T}^{-1} \cdot \hat{\tilde{m}}(z) \cdot \hat{T}$ and the nonlinear terms are

$$N_1 = c(z)(|\Psi_1|^2 + \frac{2}{3}|\Psi_2|^2)\Psi_1,$$
$$N_2 = c(z)(\frac{2}{3}|\Psi_1|^2 + |\Psi_2|^2)\Psi_2. \tag{7}$$

The typical scale $z_{corr}$ of variation of $\hat{m}$ in optical fibers is between 10m and 100m. Typical scales at which a pulse experiences essential distortion are a dispersion length $z_{disp} \equiv t_0^2/d_1$, $d_1$ is the typical amplitude of dispersion variations, $t_0$ is the typical pulse width; a nonlinear length $z_{nl} \equiv 1/(c(z)p^2)$, $p$ is the typical pulse amplitude; and PMD length $z_m \equiv t_0/m_0$, $m_0$ is the typical amplitude of variation of matrix $\hat{m}$ components. As a typical example one can set $t_0 \sim 10ps$, $d_1 \sim 1ps^2/km$, $p^2 \sim 2mW$, $c(z) \sim 0.001\,(km\,mW)^{-1}$, and $m_0 \sim 1ps/km$. One gets $z_{disp} \sim 100km$, $z_{nl} \sim 500km$, and $z_m \sim 10km$. Thus, a minimal length of pulse distortion $z_{pulse}$ is $\sim z_m \sim 10km$. the length $z_{pulse}$ is much larger than $z_{corr}$ and,

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according to the central limit theorem, $\hat{m}$ can be treated at scales of the order of $z_{pulse}$ as random Gaussian processes with zero correlation length and zero mean $\langle \hat{m} \rangle = \hat{0}$ ($\langle \ldots \rangle$ means ensemble averaging over statistics of $\hat{m}(z)$ and $d(z)$).

The traceless matrix $\hat{m}$ can be represented in terms of Pauli matrices: $\hat{m}(z) = \sum_{j=1}^{3} m_j(z)\hat{\sigma}_j$, where the correlation functions of components of the real vector $\mathbf{m}$ are given by

$$\langle m_j(z_1)m_k(z_2) \rangle = M_j \delta_{jk} \delta(z_1 - z_2). \tag{8}$$

The vector $\mathbf{M}$ does not depend on $z$ and $\mathbf{M}$ is defined from the original problem with short but nonzero correlation lengths as $M_j = \int \langle m_j(z)m_j(z') \rangle dz'$.

Similarly, dispersion $d(z)$ in a dispersion-shifted fiber (where average GVD is shifted to zero but random variations of GVD are essential [8, 9, 11]) can be represented by random Gaussian process with zero correlation length and zero mean:

$$\langle d(z_1)d(z_2) \rangle = D\delta(z_1 - z_2), \quad \langle d(z) \rangle = 0, \tag{9}$$

where $D = \int \langle d(z)d(z') \rangle dz'$. The quantities $\hat{m}(z)$ and $d(z)$ are assumed to be the independent random processes: $\langle m_j(z)d(z') \rangle = 0$.

Below the Eqs. $(6) - (9)$ are used to obtain exact result on evolution of pulse width along $z$. First one can conclude that as $\langle \hat{m}(z) \rangle = \hat{0}$ and $\langle d(z) \rangle = 0$, there is no preferred direction along $t$ and the average pulse position is zero:

$$\langle t \rangle \equiv \langle \int t(|\Psi_1|^2 + |\Psi|^2)dt \rangle / P = 0, \tag{10}$$

where

$$P = \int (|\Psi_1|^2 + |\Psi_2|^2)dt \tag{11}$$

is the time-averaged optical power. The quantity $P$ is an integral of motion of VNLS (6) : $\partial_z P_z = 0$.

Consider now evolution along $z$ of

$$A \equiv \int (t - \langle t \rangle)^2(|\Psi_1|^2 + |\Psi|^2)dt, \tag{12}$$

which is related to the root mean square pulse width $T_{RMS}$ by

$$T_{RMS}^2 = A/P. \tag{13}$$

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Using Eqs. (6) and (10), integrating by parts over $t$, and applying vanishing boundary conditions at infinity one derives

$$A_z = d(z)B^{(d)} - \sum_{j=1}^{3} m_j(z)\, B^{(m_j)}, \tag{14}$$

where

$$B^{(d)} = \int 2it \sum_{\alpha=1}^{2}(\Psi_\alpha \partial_t \Psi_\alpha^* - \Psi_\alpha^* \partial_t \Psi_\alpha)dt,$$

$$B^{(m_1)} = -2\int t(\Psi_1^* \Psi_2 + c.c.)dt,$$

$$B^{(m_2)} = 2i\int t(\Psi_1^* \Psi_2 - c.c.)dt, \tag{15}$$

$$B^{(m_3)} = -2\int t(|\Psi_1|^2 - |\Psi_2|^2)dt.$$

It is essential that all expressions in rhs of (15) do not have explicit dependence on random variables $d$ and $\mathbf{m}$, which allows one to differentiate them over $z$ and find, using again Eq. (6) and integrating by parts, that

$$B_z^{(d)} = 8d(z)X - 2c(z)Y + O(\mathbf{m}), \tag{16a}$$

$$B_z^{(m_1)} = -2m_1 P + O(d, m_2, m_3, c(z)), \tag{16b}$$

$$B_z^{(m_2)} = -2m_2 P + O(d, m_1, m_3, c(z)), \tag{16c}$$

$$B_z^{(m_3)} = -2m_3 P + O(d, m_1, m_2), \tag{16d}$$

where

$$X \equiv \int \left(|\partial_t \Psi_1|^2 + |\partial_t \Psi_2|^2\right)dt,$$

$$Y \equiv \int (|\Psi_1|^4 + |\Psi_2|^4 + \frac{4}{3}|\Psi_1|^2|\Psi_2|^2)dt. \tag{17}$$

Notation $O(d, m_2, m_3, c(z))$ in Eq. (16b) means extra terms which are linear in $d, m_2, m_3, c(z)$. And the same type of notation is used for the similar terms in rhs of Eqs. (16a), (16c), (16d). Explicit expressions for these terms are not given here because they are bulky and, as it is shown below, they vanish after statistical averaging. Note that for $\mathbf{m} = 0$, $\Psi_2 = 0$ and $d = Const$ Eq. (16a) coincides with a so-called virial theorem [23, 24, 25]. However, direct application of the virial theorem to Eq. (6) is not possible because it would require determination of $A_{zz}$ by differentiating Eq. (14) over $z$, which

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would result in appearance of terms including derivatives of random variables $d(z)$ and $\mathbf{m}$ over $z$. Here these problems are avoided by studying $B_z^{(d)}$ and $B_z^{(m_j)}$ instead of $A_{zz}$, but as a result of such procedure, one obtains cumbersome expressions $(16a) - (16d)$ compare with the compact expression for the virial theorem in deterministic scalar case (see Refs. [23, 24, 25]).

Consider the statistical average of

$$A_z^{(d)} = d(z)\Big(B^{(d)}(0) + \int_0^z B_{z'}^{(d)}(z')dz'\Big), \tag{18}$$

and substitute $B_{z'}^{(d)}(z')$ in that equation by rhs of Eq. (16a). Assume for a moment (to choose a correct limit $z_{corr} \to 0$) that the correlation length $z_{corr}$ of all random processes $d(z)$ and $\mathbf{m}(z)$ is small but finite and that all correlation functions decay at least exponentially or faster with distance. Now choose a distance $z_0$ in such a way that $z_{corr} \ll z_0 \ll z_{pulse}$. The statistical average of Eq. (18), using Eq. (16a), gives

$$\langle A_z^{(d)}\rangle = \langle d(z)\int_{z_0}^z \Big(8d(z')X(z') - 2c(z')Y(z')\Big)dz'\rangle, \tag{19}$$

where all corrections to that equation are exponentially small and vanish in the limit $z_{corr} \to 0$ due to casuality constrain. Casuality constrain implies here that $\Psi(z)$ does not depend on $d(\tilde{z})$ and $\mathbf{m}(\tilde{z})$ if $z < \tilde{z}$.

The condition $z_0 \ll z_{pulse}$ allows one to write solution of VNLS (6) as $\Psi(z') = \Psi(z_0) + O(z' - z_0)$ for $z_0 \le z' \le z$. Taking the limit $z_{corr} \to 0$, for which $z_0 \to z$, and using Eq. (19) one obtains

$$\langle A_z^{(d)}\rangle = 4D\langle X\rangle. \tag{20}$$

To average Eq. (14) one rewrites it in the equivalent form

$$A_z = d(z)\Big(B^{(d)}(0) + \int_0^z B_{z'}^{(d)}(z')dz'\Big)$$
$$- \sum_{j=1}^3 m_j(z)\Big(B^{(m_j)}(0) + \int_0^z B_{z'}^{(m_j)}(z')dz'\Big), \tag{21}$$

and substitute expressions for $B_z^{(d)}, B_z^{(m_1)}, B_z^{(m_2)}, B_z^{(m_3)}$ from rhs of system $(16a) - (16d)$ into Eq. (21). As a result, one finds, by averaging over distribution of $\mathbf{m}$ and $d$, that

$$\langle A_z\rangle = P\sum_{j=1}^3 M_j + 4D\langle X\rangle. \tag{22}$$

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Here $\langle P \rangle = P$ because $P$ is an integral o motion. All terms in rhs of Eq. (22) are positive definite and hence $\langle A \rangle$ grows with $z$.

Using Eqs. (13) and (22) one obtains an expression for the statistical average of the root mean square pulse width

$$\langle T_{RMS}(z)^2 \rangle = T_{RMS}(0)^2 + z \sum_{j=1}^{3} M_j + \frac{4D}{P} \int_0^z \langle X(z') \rangle dz'. \qquad (23)$$

Eq. (23) is exact for system $(6) - (9)$ because Eq. (23) is derived for arbitrary strong nonlinearity. Thus, this is essentially nonperturbative result. Remarkably, Eq. (23) does not explicitly depend on the nonlinear coefficient $c(z)$. This fact is very peculiar propertiy of VNLS (6) and is related both to the generalization of the virial theorem of scalar nonlinear Schrödinger equation [23, 24, 25] and to the zero correlation length limit of fiber parameters (8) and (9). In the linear case, which corresponds to $c(z) = 0$, $X$ does not depend on $z$ and the growth of $\langle T_{RMS}(z)^2 \rangle$ with $z$ is linear (i.e. the pulse width experiences diffusive growth caused by random variations of GVD and PMD matrix $\hat{m}(z)$). For nonzero $c(z)$, the nonlinearity is only responsible for nontrivial dependence of $\langle X \rangle$ on $z$, and therefore, for modification of the (still) diffusive law. The term $P \sum_{j=1}^{3} M_j$ is constant for arbitrarily strong nonlinearity which means that growth of the pulse width can not be slower than linear in distance $z$. This gives a fundamental limit to the bit-rate of information transmission in optical fiber systems.

Estimating $X$, $P$, $|\mathbf{M}|$ and $D$ by $p^2/t_0$, $p^2 t_0$, $z_{corr} m_0^2$ and $z_{corr} d_1^2$, respectively, one derives from Eq. (23) that $\beta \equiv (\langle T_{RMS}(z)^2 \rangle - T_{RMS}(0)^2)/T_{RMS}(0)^2$ $\sim z z_{corr} \left( m_0^2/t_0^2 + d_1^2/t_0^4 \right)$. Then the minimal requirement for small information loss, $\beta \stackrel{<}{\sim} 1$, results in a limitation for the pulse width: $t_0 \stackrel{>}{\sim} 10ps$, for $z \sim 10^3 km$, and the typical values $z_{corr} \sim 100m$, $m_0 \sim 1ps/km$, $d_1 \sim 1ps^2/km$. It suggests that construction of high-bit-rate lines based on the dispersion-shifted fiber requires essential improvement of fiber production technology, and/or implementation of both PMD compensation [26, 27] and the pinning method [10], to reduce both PMD and random GVD variation effects.

Note that the extension of the results of this Article to the case of both nonzero average dispersion (dispersion-shifted fiber with nonzero average dispersion) and dispersion-managed systems is an open problem. E.g. nonzero $\langle d \rangle = d_0(z)$ would result in appearance of new terms $8d_0(z) \int_{z_0}^z \left[ d_0(z') \langle X(z') \rangle - 2c(z') \langle Y(z') \rangle \right] dz'$ in rhs of Eq. (22) which generally are not sign-definite. In the case of dispersion management these terms oscillate fast with distance

$z$ so their contribution to Eq. (22) can be small. Additional research is necessary for the case of nonzero average dispersion which is outside the scope of this Article.

In conclusion, the exact analytical expression (23) for random diffusion of the averaged optical pulse width is derived. Eq. (23) introduces a fundamental limit on the minimal pulse width for which information transmission is possible in nonlinear dispersion-shifted optical fiber systems with zero average dispersion.

E-mail address: lushnikov@cnls.lanl.gov.

* Electronic address: lushnikov@cnls.lanl.gov

[1] C.D. Poole, Opt.Lett.**13**, 687 (1988)).

[2] L.F. Mollenauer, K. Smith, J.P. Gordon, C.R. Menyuk, Opt. Lett., **14**, 1219 (1989).

[3] C.R. Menyuk, IEEE J. of Quantum Electronics **QE25**, 2674 (1989).

[4] C.D. Poole, J.H. Winters, and J.A. Nagel, Opt.Lett.**16**, 372 (1991).

[5] N. Gisin and J.P. Pellax, Opt. Comm.**89**, 316 (1992)).

[6] P.K.A. Wai and C.R. Menyuk, IEEE J. of Lightwave Technology, **14**, 148 (1996).

[7] J. Yang, W.I. Kath, and C.R. Menyuk, Opt.Lett.**26**, 1472 (2001).

[8] L.F. Mollenauer, P.V. Mamyshev, M.J. Neubelt, Opt.Lett.**21**, 1724 (1996).

[9] L.F. Mollenauer, P.V. Mamyshev, J. Gripp, M.J. Neubelt, N. Mamysheva, L. Gruner-Nielsen, T. Veng, Optics Letters, **25**, 704 (2000).

[10] M. Chertkov, I. Gabitov, and J. Moeser, Proc. of the Natl. Acad of Sci., **98**, 14208 (2001).

[11] M. Chertkov, I. Gabitov, P.M. Lushnikov, J. Moeser, and Z. Toroczkai, J. of the Optical Society of America B, **19**, 2538-2550 (2002).

[12] T.I. Lakoba and D.J. Kaup, Phys. Rev. E, **56**, 6147 (1997).

[13] M. Matsumoto, Y. Akagi, and A. Hasegawa, IEEE J. of Lightwave Technology, **15**, 584 (1997).

[14] M. Chertkov, I. Gabitov, I. Kolokolov, and V. Lebedev, JETP Lett., **74**, 535 (2001).

[15] M. Chertkov, I. Gabitov, I. Kolokolov, and T. Schafer, J. of the Optical Society of America B, **21**, 486-498 (2004).

[16] F.Kh. Abdullaev, J. C. Bronski and G. Papanicolaou, Physica **D**135, 369 (2000).

[17] F.Kh.Abdullaev, B.B.Baizakov, Opt.Lett.**25**, 93 (2000).

[18] M. Chertkov, I. Gabitov, I. Kolokolov, and V. Lebedev, JETP Lett., **74**, 357 (2001).

[19] A.L. Berkhoer and V.E. Zakharov, Sov. Phys. JETP **31**, 486 (1970).

[20] G.P. Agrawal, Fiber-optic communication systems, New York, Wiley, 1997.

[21] I.R. Gabitov and S.K. Turitsyn, Opt. Lett., **21**, 327 (1996).

[22] P.M. Lushnikov, Opt.Lett.**27**, 939 (2002).

[23] V. E. Zakharov, Sov. Phys. JETP **35**, 908 (1972).

[24] P.M. Lushnikov, JETP Lett. **62**, 461 (1995).

[25] P.M. Lushnikov, JETP Lett. **72**, 111 (2000).

[26] C.J. Xie, M. Karlsson, H. Sunnerud, P.A. Andrekson, Opt. Lett., **26**, 672 (2001).

[27] H. Sunnerud, C.J. Xie, M. Karlsson, R. Samuelsson, P.A. Andrekson, IEEE J. of Lightwave Technology, **20**, 368 (2002).